Rigorous Full-Wave Analysis of Microstrip Transmission Structures

Bin Song and Junmei Fu

Abstract—**A boundary-element technique is proposed for the rigorous full-wave study of a generalized microstrip transmission configuration. The method is accurate and covers the metallization thickness, mounting grooves, and arbitrary cross sections. Comparison of the obtained results with data available in literature shows the efficiency of this approach.**

I. Introduction

FOR FULL-WAVE analysis of microstrip lines, many methods exist. However, most of the methods either have restricted application or require a large computer memory and long computing time, e.g., the modified mode-matching technique [1] and the transverse resonance method [2] can only study structures of regular cross sections, and a few numerical technique, e.g., the finite-element method (FEM) may handle arbitrary cross-sectional geometries [3], but the number of nodal points divided is very large, so the CPU time required is considerable. In this letter, we present a very general boundary-element procedure which can analyze generalized microstrip configurations of arbitrary cross sections including finite-metallization thickness, substrated mounting grooves, asymmetric structures, and more than one dielectric region. The boundary-element method (BEM) with constant elements has been applied to calculating dispersion characteristics of finlines by Song and Fu [4]. In this contribution, the BEM with line elements is described for the full-wave investigation of generalized microstrip transmission structures.

II. Theory

Consider a waveguide of arbitrary shape that is uniform in the Z direction and that consists of isotropic, lossless dielectric media. The cross section can be divided into several subregions homogeneously filled with a dielectric material. Assumed that the electromagnetic wave propagates along the Z direction of the form $e^{j(\omega t-\beta z)}$ and in a typical subregion S_i, the longitudinal field components satisfy the Helmholtz's equation

$$\nabla_t^2 E_z + k_c^2 E_z = 0, \tag{1a}$$

$$\nabla_t^2 H_z + k_c^2 H_z = 0, \tag{1b}$$

where ∇_t^2 is the transverse Laplacian operator, and k_c is the cut-off wavenumber, namely, $k_c^2 = k_i^2 - \beta^2$, here k_i is the wavenumber in the subregion. Using the method of weighted residuals [5], from (1) the following equations can be obtained,

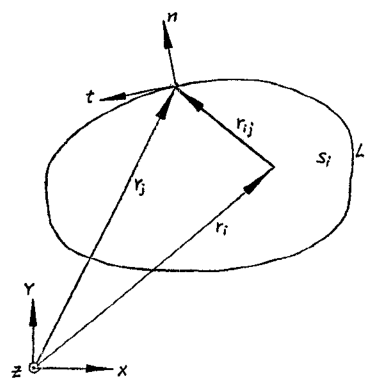

Fig. 1. Two-dimensional region surrounded by boundary L.

$$E_z(\boldsymbol{r_i}) + \int_L \frac{\partial G(\boldsymbol{r_i},\boldsymbol{r_j})}{\partial n} E_z(\boldsymbol{r_j})\,dl = \int_L G(\boldsymbol{r_i}\boldsymbol{r_j})\frac{\partial E_z(\boldsymbol{r_j})}{\partial n}\,dl \tag{2a}$$

$$H_z(\boldsymbol{r_i}) + \int_L \frac{\partial G(\boldsymbol{r_i},\boldsymbol{r_j})}{\partial n} H_z(\boldsymbol{r_j})\,dl = \int_L G(\boldsymbol{r_i}\boldsymbol{r_j})\frac{\partial H_z(\boldsymbol{r_j})}{\partial n}\,dl. \tag{2b}$$

Here, $G(\boldsymbol{r_i},\boldsymbol{r_j})$ is Green's function in two-dimensional free-space. The position of a nodal point inside subregion is represented by the vector $\boldsymbol{r_i}$, and that on boundary L is denoted by the vector $\boldsymbol{r_j}$. $\boldsymbol{n}$ and $\boldsymbol{t}$ refer to the outward unit normal vector and the tangential unit normal vector, respectively, as shown in Fig. 1.

Assumed that the boundary L is smooth, letting $\boldsymbol{r_i}$ approaches the boundary and considering Gauss' principle and Cauchy's principal value of integration, and using the following equations

$$E_t = \frac{j\omega\mu}{k_c^2}\left(\frac{\partial H_z}{\partial n} - \frac{\beta}{\omega\mu}\cdot\frac{\partial E_z}{\partial t}\right)$$

$$H_t = -\frac{j\omega\varepsilon_i}{k_c^2}\left(\frac{\partial E_z}{\partial n} + \frac{\beta}{\omega\varepsilon_i}\cdot\frac{\partial H_z}{\partial t}\right). \tag{3}$$

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The authors are with the Department of Information and Control Engineering, Xi'an Jiatong University, Shaanxi 710049, P.R.C.
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Equation (2) can be rewritten as

$$\left(\frac{1}{2}\right)E_z(\boldsymbol{r_i}) + \int_L \frac{\partial G(\boldsymbol{r_i}, \boldsymbol{r_j})}{\partial n} E_z(\boldsymbol{r_j})\, dl = \int_L G(\boldsymbol{r_i r_j}) - \frac{k_c^2}{j\omega\varepsilon_i} H_t(\boldsymbol{r_j})\, dl + \frac{\beta}{\omega\varepsilon_i}\int_L \frac{\partial G(\boldsymbol{r_i}, \boldsymbol{r_j})}{\partial t} H_z(\boldsymbol{r_j})\, dl \quad (4a)$$

$$\left(\frac{1}{2}\right)H_z(\boldsymbol{r_i}) + \int_L \frac{\partial G(\boldsymbol{r_i}, \boldsymbol{r_j})}{\partial n} H_z(\boldsymbol{r_j})\, dl = \int_L G(\boldsymbol{r_i r_j}) \frac{k_c^2}{j\omega\mu} E_t(\boldsymbol{r_j})\, dl - \frac{\beta}{\omega\mu}\int_L \frac{\partial G(\boldsymbol{r_i}, \boldsymbol{r_j})}{\partial t} E_z(\boldsymbol{r_j})\, dl. \quad (4b)$$

Dividing the boundary L into N line elements, and (4) is then discretized as

$$\left(\frac{1}{2}\right)E_{z_i} + \sum_{l=1}^{N.} [a_1, a_2]_l \begin{bmatrix} E_{z_1} \\ E_{z_2} \end{bmatrix}_l = \frac{k_c^2}{j\omega\varepsilon_i}\sum_{l=1}^{N} [b_1, b_2]_l \begin{bmatrix} H_{t_1} \\ H_{t_2} \end{bmatrix}_l + \frac{\beta}{\omega\varepsilon_i}\sum_{l=1}^{N} [c_1, c_2]_l \begin{bmatrix} H_{z_1} \\ H_{z_2} \end{bmatrix}_l, \quad (5a)$$

$$\left(\frac{1}{2}\right)H_{z_i} + \sum_{l=1}^{N} [a_1, a_2]_l \begin{bmatrix} H_{z_1} \\ H_{z_2} \end{bmatrix}_l = -\frac{k_c^2}{j\omega\mu}\sum_{l=1}^{N} [b_1, b_2]_l \begin{bmatrix} E_{t_1} \\ E_{t_2} \end{bmatrix}_l - \frac{\beta}{\omega\mu}\sum_{l=1}^{N} [c_1, c_2]_l \begin{bmatrix} E_{z_1} \\ E_{z_2} \end{bmatrix}_l, \quad (5b)$$

where $H_{zj}, E_{zj}, H_{tj}, H_{tj} (j = 1, 2)$are the field quantities on the two nodes of *e*th element. The coefficients $a_j, b_j, c_j (j = 1, 2)$ are contributions of the *e*th element to integration, and they may be calculated with Gaussian integration. Equation (5) can be represented in the following matrix form:

$$[A][E_z] + \frac{k_c^2}{j\omega\varepsilon_i}[B][H_t] - \frac{\beta}{\omega\varepsilon_i}[C][Hz] = 0 \quad (6a)$$

$$[A][H_z] - \frac{k_c^2}{j\omega\mu}[B][H_t] + \frac{\beta}{\omega\mu}[C][E_z] = 0. \quad (6b)$$

In the same way for subregion S_j, the boundary matrix equation similar to (6) can be formed. Considering the boundary conditions along the common interface between contiguous subregions, namely, the continuity of the tangential electric and magnetic fields, it is required the following equations,

$$E_z^{(i)} = E_z^{(j)}; \quad H_z^{(i)} = H_z^{(j)};$$
$$E_t^{(i)} = E_t^{(j)}; \quad H_t^{(i)} = H_t^{(j)}. \quad (7)$$

So the final boundary matrix equation of the whole region can be obtained, and can be written as a homogeneous equations set in the form

$$[U][X] = 0, \quad (8)$$

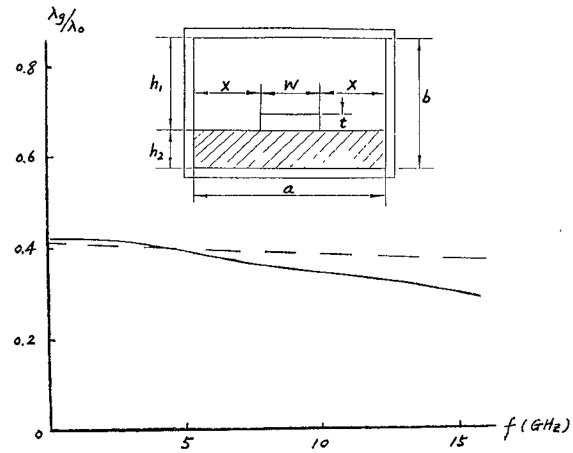

Fig. 2. Dispersion characteristics of a shielded microstrip line: ($a = b =$ 12.7 mm; ×1 = ×2 = 5.715 mm; c = 1 27 mm; t = 0.005 mm; $\varepsilon_r = 8.875$) ———— $t = 0.005$ mm; — — — — $t = 0$.

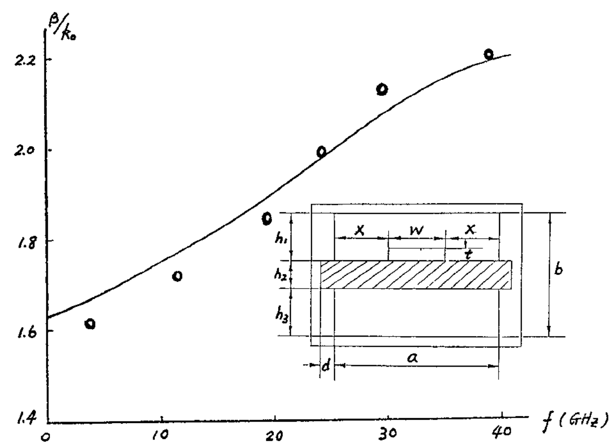

Fig. 3. Dispersion characteristics of the suspended microstrip line: ($a = 2b = 7.112$ mm; $h_2 = 0.635$ mm; $h_1 = h_3 = 1.4605$ mm; $w = 1.0$ mm; $t = 0.005$ mm; $e = 0.01$ mm; $\varepsilon_r = 9.6$) oooo $t = e = 0$; ———— $t = 0.005$ mm, $e = 0.01$ mm.

where the elements of matrix $[U]$ are all the functions of phase constant β and $[X]$ contains the unknown E_z, H_z, E_t and H_t on the boundary. Considering the condition that makes (7) exist nonzero solutions, namely,

$$|U| = 0, \quad (9)$$

the propagation constant β can be obtained.

In this process, $-(j/4)H_o^{(2)}(k_c\ r_{ij})$, which satisfies the radiation condition, is employed as Green's function, so no spurious solutions appear. In addition, inhomogeneous discretization is used to enhance the accuracy of the results obtained.

III. Results

First of all, a shielded microstrip line treated with the mode-matching technique in [1] is calculated with our method. In [1], the finite-strip thickness is negligible, but in our calculation, its effects is taken into account. From the obtained results, as shown in Fig. 2, we can find that the propagation constants of the fundamental mode at high frequencies slightly decrease because the electromagnetic field is mainly concentrated between the strip and the upper waveguide wall.

In order to show the flexibility of this method, a suspended microstrip structure is studied. Calculated results for the dispersion characteristics are shown in Fig. 3. The obtained results for zero grooves depth and zero metallization thickness are in agreement with data obtained via the transverse resonance method [2]. For a finite-grooves depth and finite-metallization thickness, the propagation constant of the fundamental mode decreases. These BEM calculations are performed with 118 nodes, while the corresponding FEM calculations need at least 400 nodes.

IV. Conclusion

In this letter, a boundary-element procedure for the rigorous investigation of generalized microstrip structures is proposed, the method has a few merits, i.e., it can handle generalized microstrip configuration that includes finite-strip thickness, substrate mounting grooves, more than one dielectric region and arbitrary cross-sectional geometries, moreover, it only requires a small computer memory and short computation time and the results obtained have fairly good accuracy. In addition, this approach can be applied to the full-wave analysis of various microwave and millimeter-wave transmission lines with arbitrary cross-sectional geometries.

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